Poker Probability from Wikipedia

Frequency of 5-card poker hands

The following enumerates the frequency of each hand, given all combinations of 5 cards randomly drawn from a full deck of 52 without replacement. Wild cards are not considered. The probability of drawing a given hand is calculated by dividing the number of ways of drawing the hand by the total number of 5-card hands (the sample space, \( \binom{52}{5} = 2,598,960 \)) (five-card hands). The odds are defined as the ratio \((1/p) - 1 : 1\), where \(p\) is the probability. (The frequencies given are exact; the probabilities and odds are approximate.)

Please note, that in the interests of calculating these values for yourselves, the function \( nCr \) on most scientific calculators can be used. To see what the actual formula looks like, please see the And five card poker hand below.

<table>
<thead>
<tr>
<th>Visual help</th>
<th>Hand</th>
<th>Frequency</th>
<th>Probability</th>
<th>Cumulative</th>
<th>Odds</th>
<th>Mathematical expression of frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Royal flush image]</td>
<td>Royal flush</td>
<td>4</td>
<td>0.000154%</td>
<td>0.000154%</td>
<td>649,739 : 1</td>
<td>( \binom{4}{1} )</td>
</tr>
<tr>
<td>![Straight flush image]</td>
<td>Straight flush (excluding royal flush)</td>
<td>36</td>
<td>0.00139%</td>
<td>0.00154%</td>
<td>72,192.33 : 1</td>
<td>( \binom{10}{1} \binom{4}{1} - \binom{4}{1} )</td>
</tr>
<tr>
<td>![Four of a kind image]</td>
<td>Four of a kind</td>
<td>624</td>
<td>0.0240%</td>
<td>0.0256%</td>
<td>4,164 : 1</td>
<td>( \binom{13}{1} \binom{4}{1} \binom{48}{1} )</td>
</tr>
<tr>
<td>![Full house image]</td>
<td>Full house</td>
<td>3,744</td>
<td>0.144%</td>
<td>0.170%</td>
<td>693.2 : 1</td>
<td>( \binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} )</td>
</tr>
<tr>
<td>Hand</td>
<td>Frequency</td>
<td>Percent</td>
<td>Cumulative Percent</td>
<td>Odds</td>
<td>Combination Formula</td>
<td></td>
</tr>
<tr>
<td>-------------</td>
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<td></td>
</tr>
<tr>
<td>Flush</td>
<td>5,108</td>
<td>0.197%</td>
<td>0.367%</td>
<td>507.8 : 1</td>
<td>( \binom{13}{5} \binom{4}{1} - \binom{10}{1} \binom{4}{1} )</td>
<td></td>
</tr>
<tr>
<td>Straight</td>
<td>10,200</td>
<td>0.392%</td>
<td>0.76%</td>
<td>253.8 : 1</td>
<td>( \binom{10}{1} \binom{4}{5} - \binom{10}{1} \binom{4}{1} )</td>
<td></td>
</tr>
<tr>
<td>Three of a kind</td>
<td>54,912</td>
<td>2.11%</td>
<td>2.87%</td>
<td>46.3 : 1</td>
<td>( \binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{2} )</td>
<td></td>
</tr>
<tr>
<td>Two pair</td>
<td>123,552</td>
<td>4.75%</td>
<td>7.62%</td>
<td>20.03 : 1</td>
<td>( \binom{13}{2} \binom{4}{2} \binom{11}{1} \binom{4}{1} )</td>
<td></td>
</tr>
<tr>
<td>One pair</td>
<td>1,098,240</td>
<td>42.3%</td>
<td>49.9%</td>
<td>1.37 : 1</td>
<td>( \binom{13}{1} \binom{4}{2} \binom{12}{3} \binom{4}{3} )</td>
<td></td>
</tr>
<tr>
<td>No pair / High card</td>
<td>1,302,540</td>
<td>50.1%</td>
<td>100%</td>
<td>0.995 : 1</td>
<td>( \left[ \binom{13}{5} - 10 \right] \left[ \binom{4}{5} - 4 \right] )</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>2,598,960</td>
<td>100%</td>
<td>100%</td>
<td>0 : 1</td>
<td>( \binom{52}{5} )</td>
<td></td>
</tr>
</tbody>
</table>
The royal flush is a case of the straight flush. It can be formed 4 ways (one for each suit), giving it a probability of 0.000154% and odds of 649,739 : 1.

When ace-low straights and ace-low straight flushes are not counted, the probabilities of each are reduced: straights and straight flushes each become 9/10 as common as they otherwise would be. The 4 missed straight flushes become flushes and the 1,020 missed straights become no pair.

Note that since suits have no relative value in poker, two hands can be considered identical if one hand can be transformed into the other by swapping suits. For example, the hand 3♣ 7♣ 8♣ Q♠ A♠ is identical to 3♦ 7♦ 8♦ Q♥ A♥ because replacing all of the clubs in the first hand with diamonds and all of the spades with hearts produces the second hand. So eliminating identical hands that ignore relative suit values, there are only 134,459 distinct hands.

The number of distinct poker hands is even smaller. For example, 3♣ 7♣ 8♣ Q♠ A♠ and 3♦ 7♦ 8♦ Q♥ A♥ are not identical hands when just ignoring suit assignments because one hand has three suits, while the other hand has only two—that difference could affect the relative value of each hand when there are more cards to come. However, even though the hands are not identical from that perspective, they still form equivalent poker hands because each hand is an A-Q-8-7-3 high card hand. There are 7,462 distinct poker hands.

**Derivation of frequencies of 5-card poker hands**

The following computations show how the above frequencies for 5-card poker hands were determined. To understand these derivations, the reader should be familiar with the basic properties of the binomial coefficients and their interpretation as the number of ways of choosing elements from a given set. See also: sample space and event (probability theory).

- **Straight flush** — Each straight flush is uniquely determined by its highest ranking card; and these ranks go from 5 (A-2-3-4-5) up to A (10-J-Q-K-A) in each of the 4 suits. Thus, the total number of straight flushes is:

\[
\binom{10}{1} \binom{4}{1} = 40
\]

- **Royal straight flush** — A royal straight flush is a subset of all straight flushes in which the ace is the highest card (ie 10-J-Q-K-A in any of the four suits). Thus, the total number of royal straight flushes is

\[
\binom{5}{1} \binom{8}{1} \binom{4}{1} = 4
\]

or simply \(\binom{4}{1} = 4\). Note: this means that the total number of non-Royal straight flushes is 36.
- **Four of a kind** — Any one of the thirteen ranks can form the four of a kind by selecting all four of the suits in that rank, leaving $52 - 4 = 48$ possibilities for the final card. Thus, the total number of four-of-a-kinds is:

$$\binom{13}{1} \binom{4}{4} \binom{48}{1} = 624$$

- **Full house** — The full house comprises a triple (three of a kind) and a pair. The triple can be any one of the thirteen ranks, and three of the four cards of this rank are chosen. The pair can be any one of the remaining twelve ranks, and two of the four cards of the rank are chosen. Thus, the total number of full houses is:

$$\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} = 3,744$$

- **Flush** — The flush contains any five of the thirteen ranks, all of which belong to one of the four suits, minus the 40 straight flushes. Thus, the total number of flushes is:

$$\binom{13}{4} - 40 = 5,108$$

- **Straight** — The straight consists of any one of the ten possible sequences of five consecutive cards, from 5-4-3-2-A to A-K-Q-J-10. Each of these five cards can have any one of the four suits. Finally, as with the flush, the 40 straight flushes must be excluded, giving:

$$\binom{10}{4} - 40 = 10,200$$

- **Three of a kind** — Any of the thirteen ranks can form the three of a kind, which can contain any three of the four suits. The other cards can have any two of the remaining twelve ranks, and each can have any one of the four suits. Thus, the total number of three-of-a-kinds is:

$$\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1}^2 = 54,912$$
- **Two pair** — The pairs can have any two of the thirteen ranks, and each pair can have two of the four suits. The final card can have any one of the eleven remaining ranks, and any suit. Thus, the total number of two-pairs is:

\[
\binom{13}{2} \cdot 4^2 \cdot \binom{11}{1} \cdot 4 = 123,552
\]

- **Pair** — The pair can have any one of the thirteen ranks, and any two of the four suits. The remaining three cards can have any three of the remaining twelve ranks, and each can have any of the four suits. Thus, the total number of pair hands is:

\[
\binom{13}{1} \cdot 4 \cdot \binom{12}{3} \cdot 4^3 = 1,098,240
\]

- **No pair** — A no-pair hand contains five of the thirteen ranks, discounting the ten possible straights, and each card can have any of the four suits, discounting the four possible flushes. Alternatively, a no-pair hand is any hand that does not fall into one of the above categories; that is, any way to choose five out of 52 cards, discounting all of the above hands. Thus, the total number of no-pair hands is:

\[
\left[ \binom{13}{5} - 10 \right] \left[ \binom{4}{1}^5 - 4 \right] = \binom{52}{5} - 1,296,420 = 1,302,540
\]

- **Any five card poker hand** — The total number of five card hands that can be drawn from a deck of cards is found using a combination selecting five cards, in any order where \( n \) refers to the number of items that can be selected and \( r \) to the sample size:

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!} = \binom{52}{5} = \frac{52!}{5!(52-5)!} = 2,598,960
\]